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Phase of a Bose-Einstein Condensate in a Double-Well Potential Modulated Periodically in Time

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By using of the invariant theory, we have studied phase of a Bose-Einstein condensate in a double-well potential modulated periodically in time when the on-site interaction energy of a single pair of bosons occupying the same well equals the collision energy between two condensates, the dynamical and geometric phases are presented respectively. The Aharonov-Anandan phase is also obtained in the case of the cyclical evolution.

KEY WORDS: phase; Bose-Einstein condensation.

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1. INTRODUCTION

Recently, much attention has been paid to the investigation of Bose-Einstein condensation (BEC) in dilute and ultracold gases of neutral alkali-metal atoms using a combination of laser and evaporative cooling (Anderson *et al.*, 1995; Bradley *et al.*, 1995; Davis *et al.*, 1995; Mewes *et al.*, 1996a,b, 1997) due to the optical properties (Javanainen, 1994, 1995a,b, 1996; Lewenstein and You, 1993; Lewenstein *et al.*, 1994; Politzer, 1991, 1997; You *et al.*, 1994, 1995), statistical properties (Chou *et al.*, 1996, 1997; Grossman and Holthans, 1995a; Kuang, 1998; Stoof, 1994; Timmermans *et al.*, 1997; Yu and Jiao, 2001), phase properties (Castin and Dalibard, 1997; Cirac *et al.*, 1996; Imamoglu and Kennedy, 1997;

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Jack *et al*., 1996; Javanainen, 1994, 1995a,b, 1996; Javanainen and Ruostekoski, 1995; Javanainen and Yoo, 1996; Javanainen and Wilkens, 1997; Ruostekoski and Walls, 1997a,b; Wong *et al*., 1996; Yu and Jiao, 2003; **?**; Zou *et al*., 2002), and tunneling effect (Eckardt *et al*., 2005; Grossman and Holthans, 1995b; Jack *et al*., 1997; Javanainen, 1986, 1991; Kuang and Ouyang, 2000; Liang *et al*., 2003; Li *et al*., 2001; Liu *et al*., 2000, 2002; Milburn *et al*., 1977; Niu *et al*., 1999; Yu and Jiao, 2001; Wu, 1996; Wu *et al*., 2000, 2001, 2006). In a recent experiment, it has become possible to monitor the Josephson-like oscillation of a sample of about 1000 Bose-Einstein condensed atoms in an optical double-well potential and to observe *in situ* both the evolution of the atomic densities and of the relative phase between the condensates in both wells (Albiez *et al*., 2005). Eckardt *et al*. (2005) have extended such experiments by modulating the trapping potential periodically in time.

As we known that the quantum invariant theory proposed by Lewis and Riesenfeld (1969) is a powerful tool for treating systems with time-dependent Hamiltonians. It was generalized in Gao *et al*. (1991) by introducing the concept of basic invariants and used to study the geometric phases (Aharonov and Anandan, 1987; Berry, 1984; Simon, 1983) in connection with the exact solutions of the corresponding time-dependent Schrödinger equations. The discovery of Berry's phase is not only a breakthrough in the older theory of quantum adiabatic approximations (Berry, 1984; Simon, 1983), but also provides us with new insights in many physical phenomena. The concept of Berry's phase has developed in some different directions (Mead, 1987; Moody *et al*., 1986; Richardson *et al*., 1988; Sun, 1990, 1993, 1988a,b; Sun *et al*., 2001; Wilczek and Zee, 1984).

In this paper, by using of the invariant theory ,we shall study the dynamical and the geometric phases of a Bose-Einstein condensate in a double-well potential modulated periodically in time.

2. MODEL

We consider a Bose-Einstein condensate modulated by a double-well potential in time, the Hamiltonian of such a system can be written by (Eckardt *et al*., 2005) (in the unit of$\hbar = 1$)

$$\hat{H} = -\frac{\Omega}{2}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \kappa(\hat{a}_1^{\dagger 2}\hat{a}_1^2 + \hat{a}_2^{\dagger 2}\hat{a}_2^2) + (\mu_0 + \mu_1 \sin\omega t)(\hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1) + \chi \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2, \tag{1}$$

where $\hat{a}_i$ ($\hat{a}_i^\dagger$) is the annihilation (creation) operator for a boson in the ith well, satisfying the commutation relation $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$ ($i, j = 1, 2$). The on-site interaction energy of a single pair of bosons occupying the same well is 2κ, proportional to the s-wave scattering length of the particle species. In Eq. (1), we have assumed that the two wells are tilted such that their bottoms are misaligned in energy by an amount $2\mu_0$, which should be sufficiently large compared to the tunnel splitting Ω, so that the usual Josephson oscillations are strongly suppressed. In addition, we have proposed to modulate this tilted double-well periodically in time, such that the individual wells are shifted sinusoidally up and down, in phase opposition to each other and with angular frequency ω, by an amount μ_1. Such a modulation of an optical double-well can be achieved by periodically shifting the focus of a blue-detuned laser, which creates the barrier between the two wells, with the help of a piezo-actuated mirror (Albiez *et al.*, 2005). The last term in Eq. (1) describes the collision between the atoms of the two condensates.

3. GEOMETRIC AND DYNAMICAL PHASES OF A BOSE–EINSTEIN CONDENSATE IN A DOUBLE-WELL POTENTIAL MODULATED PERIODICALLY IN TIME

For self-consistent, we first illustrate the Lewis-Riesenfeld (L-R) invariant theory (Lewis and Riesenfeld, 1969). For a one-dimensional system whose Hamiltonian $\hat{H}(t)$ is time-dependent, then there exists an operator $\hat{I}(t)$ called invariant if it satisfies the equation

$$i\frac{\partial \hat{I}(t)}{\partial t} + [\hat{I}(t), \hat{H}(t)] = 0. \tag{2}$$

The eigenvalue equation of the time-dependent invariant $|\lambda_n, t>$ is given

$$\hat{I}(t)|\lambda_n, t\rangle = \lambda_n|\lambda_n, t\rangle, \tag{3}$$

where $\frac{\partial \lambda_n}{\partial t} = 0$. The time-dependent Schrödinger equation for this system is

$$i\frac{\partial|\psi(t)\rangle_s}{\partial t} = \hat{H}(t)|\psi(t)\rangle_s. \tag{4}$$

According to the L-R invariant theory, the particular solution $|\lambda_n, t\rangle_s$ of Eq. (4) is different from the eigenfunction $|\lambda_n, t\rangle$ of $\hat{I}(t)$ only by a phase factor $\exp[i\delta_n(t)]$, i.e.,

$$|\lambda_n, t\rangle_s = \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{5}$$

which shows that $|\lambda_n, t\rangle_s$ $(n = 1, 2, \ldots)$ forms a complete set of the solutions of Eq. (4). Then the general solution of the Schrödinger equation (4) can be written by

$$|\psi(t)\rangle_s = \sum_n C_n \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{6}$$

where

$$\delta_n(t) = \int_0^t dt' \langle \lambda_n, t' | i \frac{\partial}{\partial t'} - \hat{H}(t') | \lambda_n, t' \rangle, \tag{7}$$

and $C_n = \langle \lambda_n, 0 | \psi(0) \rangle_s$.

For simplicity, we consider the case of $\chi = 2\kappa$, namely, the on-site interaction energy of a single pair of bosons occupying the same well equals the collision energy between two condensates. Then Eq. (1) becomes

$$\hat{H} = -\frac{\Omega}{2}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \kappa(\hat{N}_1^2 + \hat{N}_2^2 + 2\hat{N}_1\hat{N}_2) \\ - \kappa(\hat{N}_1 + \hat{N}_2) + (\mu_0 + \mu_1 \sin\omega t)(\hat{N}_2 - \hat{N}_1), \tag{8}$$

where $\hat{N}_i = \hat{a}_i^\dagger \hat{a}_i$ $(i = 1, 2)$. It is easy to find that $\hat{I}_1(t) = \hat{N}_1^2 + \hat{N}_2^2 + 2\hat{N}_1\hat{N}_2$ is a special invariant of this system and satisfies $\hat{I}_1(t)|m\rangle_{a_1}|n\rangle_{a_2} = \lambda_{mn}|m\rangle_{a_1}|n\rangle_{a_2}$, where $\hat{a}_1^\dagger \hat{a}_1 |m\rangle_{a_1} = m|m\rangle_{a_1}$, $\hat{a}_2^\dagger \hat{a}_2 |n\rangle_{a_2} = n|n\rangle_{a_2}$, and $\lambda_{mn} = m^2 + n^2 + 2mn$.

In the following, we can restrict the space to be in the sub-space of the eigenstate of the invariant $\hat{I}_1(t)$. Correspondingly, $\hat{I}_1(t)$ which appears in Eq. (8) can be replaced by its eigenvalue λ_{mn}.

In order to obtain the exact solutions of Eq. (4), we can define operators $\hat{K}_+$, $\hat{K}_-$ and $\hat{K}_0$ as follows:

$$\hat{K}_+ = \hat{a}_1^\dagger \hat{a}_2, \quad \hat{K}_- = \hat{a}_2^\dagger \hat{a}_1, \quad \hat{K}_0 = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2, \tag{9}$$

which hold the commutation relations

$$[\hat{K}_0, \hat{K}_\pm] = \pm 2\hat{K}_\pm, \quad [\hat{K}_+, \hat{K}_-] = \hat{K}_0, \tag{10}$$

it is easy to prove that operators $\hat{K}_+$, $\hat{K}_-$ and $\hat{K}_0$ together with the Hamiltonian $\hat{H}$ construct a quasi-algebra.

Then we can get the L-R invariant as follows

$$\hat{I}_2(t) = \cos\theta \hat{K}_0 - e^{-i\varphi} \sin\theta \hat{K}_+ - e^{i\varphi} \sin\theta \hat{K}_-, \tag{11}$$

it is apparent that $[\hat{I}_1(t), \hat{I}_2(t)] = 0$. Here θ and φ are determined by Eq. (2) and satisfy the relations

$$\dot{\theta} = -\Omega \sin\varphi, \tag{12}$$

$$\dot{\theta}\cos\theta \sin\varphi + \dot{\varphi}\sin\theta\cos\varphi + \Omega\cos\theta + 2(\mu_0 + \mu_1 \sin\omega t)\sin\theta\cos\varphi = 0, \tag{13}$$

$$\dot{\theta}\cos\theta\cos\varphi - \dot{\varphi}\sin\theta\sin\varphi - 2(\mu_0 + \mu_1 \sin\omega t)\sin\theta\sin\varphi = 0, \tag{14}$$

where dot denotes the time derivative.

We can construct the unitary transformation

$$\hat{V}(t) = \exp[\sigma \hat{K}_+ - \sigma^* \hat{K}_-], \tag{15}$$

where $\sigma = \frac{\theta}{2}e^{-i\varphi}$ and $\sigma^* = \frac{\theta}{2}e^{i\varphi}$. The invariant $\hat{I}_2(t)$ can be transformed into a new time-independent operator $\hat{I}_V$:

$$\hat{I}_V = \hat{V}^\dagger(t)\hat{I}_2(t)\hat{V}(t) = \hat{K}_0. \tag{16}$$

Correspondingly, we can get the eigenvalue equation of operator $\hat{I}_V(t)$

$$\hat{I}_V|m\rangle_{a_1}|n\rangle_{a_2} = (m-n)|m\rangle_{a_1}|n\rangle_{a_2}, \tag{17}$$

In terms of the unitary transformation $\hat{V}(t)$ and the Baker–Campbell–Hausdoff formula (Wei and Norman, 1963)

$$\hat{V}^\dagger(t)\frac{\partial \hat{V}(t)}{\partial t} = \frac{\partial \hat{L}}{\partial t} + \frac{1}{2!}\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right] + \frac{1}{3!}\left[\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right], \hat{L}\right] + \frac{1}{4!}\left[\left[\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right], \hat{L}\right], \hat{L}\right] + \cdots, \tag{18}$$

where $\hat{V}(t) = \exp[\hat{L}(t)]$, one has

$$\begin{aligned}\hat{H}_V(t) &= \hat{V}^\dagger(t)\hat{H}(t)\hat{V}(t) - i\hat{V}^\dagger(t)\frac{\partial \hat{V}(t)}{\partial t}\\ &= \kappa\lambda_{mn} + \left[\frac{\Omega}{2}\sin\theta\cos\varphi - \kappa - (\mu_0 + \mu_1\sin\omega t)\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)\right.\\ &\quad \left. + \frac{\dot{\varphi}}{2}(1-\cos\theta)\right]\hat{a}_1^\dagger\hat{a}_1 + \left[-\frac{\Omega}{2}\sin\theta\cos\varphi - \kappa + (\mu_0 + \mu_1\sin\omega t)\right.\\ &\quad \left. \times\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) - \frac{\dot{\varphi}}{2}(1-\cos\theta)\right]\hat{a}_2^\dagger\hat{a}_2,\end{aligned} \tag{19}$$

where λ_{mn} is the eigenvalue of operator $\hat{I}_1(t)$. It is easy to find that $\hat{H}(t)$ differs from $\hat{I}_V$ only by a time-dependent c-number factor. Thus we can get the general solution of the time-dependent Schrödinger equation Eq. (4)

$$|\Psi(t)\rangle_s = \sum_m\sum_n C_{mn}\exp[i\delta_{mn}(t)]\hat{V}(t)|m\rangle_{a_1}|n\rangle_{a_2}, \tag{20}$$

with the coefficients $C_{mn} = \langle m, n, t=0|\Psi(0)\rangle_s$. The phase $\delta_{mn}(t) = \delta_{mn}^d(t) + \delta_{mn}^g(t)$ includes the dynamical phase

$$\begin{aligned}\delta_{mn}^d(t) &= m\int_{t_0}^t\left[-\frac{\Omega}{2}\sin\theta\cos\varphi + \kappa + (\mu_0+\mu_1\sin\omega t)\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)\right]dt'\\ &\quad - \int_{t_0}^t \kappa\lambda_{mn}dt' + n\int_{t_0}^t\left[\frac{\Omega}{2}\sin\theta\cos\varphi + \kappa - (\mu_0+\mu_1\sin\omega t)\right.\\ &\quad \left.\times\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)\right]dt',\end{aligned} \tag{21}$$

and the geometric phase

$$\delta^g_{mn}(t) = (n - m)\int_{t_0}^{t} \frac{\dot{\varphi}}{2}(1 - \cos\theta)dt'. \tag{22}$$

In particular, the geometric phase becomes in the case of the cyclical evolution

$$\delta^g_{mn}(t) = \frac{1}{2}(n - m)\oint(1 - \cos\theta)d\varphi, \tag{23}$$

which is the geometric Aharonov-Anandan phase.

4. CONCLUSIONS

In conclusion, by using of the L-R invariant theory, we have studied phase of a Bose-Einstein condensate in a double-well potential modulated periodically in time when the on-site interaction energy of a single pair of bosons occupying the same well equals the collision energy between two condensates, the dynamical and geometric phases are presented respectively. The Aharonov-Anandan phase is obtained in the case of the cyclical evolution .

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